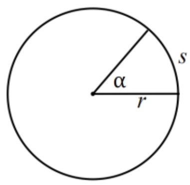
# Applications of Radian Measure

**Arc Length**

Often, we want to find the arc length ($s$) on a circle of radius $r$, intercepted by an angle $\alpha$. We can do this by determining what fraction of the circle we are looking at, then multiplying by the circumference of the circle.

**Degrees:** $s = \dfrac{\alpha}{360°} \cdot 2\pi r$

**Radians:** $s = \dfrac{\alpha}{2\pi} \cdot 2\pi r$ or $\boldsymbol{s = \alpha r}$ **This formula only works if $\boldsymbol{\alpha}$ is in radians!**




**Examples:**

A central angle of $\pi/2$ intercepts an arc on the surface of the earth that runs from the equator to the North Pole. Using 3950 miles as the radius of the earth, find the length of the intercepted arc to the nearest mile.

A wagon wheel has a diameter of 28 inches and an angle of 30° between the spokes. What is the length of the arc $s$ (to the nearest hundredth of an inch) between two adjacent spokes?

**Area of a Sector of a Circle**

Finding the area ($A$) of a sector of a circle of radius $r$ with central angle $\alpha$, is similar to finding the arc length: Determine what fraction of the circle the sector makes up, then multiply by the area of the circle.

**Degrees:** $A = \dfrac{\alpha}{360°} \cdot \pi r^2$

**Radians:** $A = \dfrac{\alpha}{2\pi} \cdot \pi r^2$ or $\boldsymbol{A = \dfrac{\alpha r^2}{2}}$ **This formula only works if $\alpha$ is in radians!**

**Examples:**

Which is bigger: a slice of pizza from a 10″ diameter pizza cut into 6 slices, or a slice from a 12″ diameter pizza cut into 8 slices?

A center-pivot irrigation system is used to water a circular field with radius 200 feet. In three hours the system waters a sector with a central angle of $\pi/8$. What area (in square feet) is watered in that time?

**Angular and Linear Velocity (Speed)**

**Velocity:** The rate at which the location of an object is changing with respect to time.

**Angular Velocity:** The rate at which the central angle is changing for an object moving in a circle. If a point is in motion on a circle through an angle of $\theta$ radians in time $t$, then its angular velocity $\omega$ is given by $\omega = \dfrac{\theta}{t}$.

Angular velocity is usually expressed as radians per unit of time (radians/hr, radians/min, radians/sec, etc.)

**Examples:**

Convert 650 rpm (revolutions per minute) to radians per minute.

(Use the fact that $1 \text{ revolution} = 2\pi \text{ radians}$).

Convert the angular velocity of 1600 rad/hr to rev/hr.

A 24-inch lawnmower blade rotates at a rate of 2000 rpm. What is the angular velocity in radians per second of a point on the tip of the blade?

A particle is moving in a circular path with a radius of 9 ft. at 30 radians per minutes. How fast is the particle rotating in revolutions per second?

**Linear Velocity:** The rate at which the position of the object is changing with respect to time. If a point is in motion on a circle of radius *r* through an angle of $\theta$ radians in time *t*, then its linear velocity *v* is given by $v = \frac{s}{t}$, where *s* is the arc length determined by $s = \theta r$.

**Examples:**
A propeller with a radius of 1.6 meters is rotating at 1500 revolutions per minute. What is the linear velocity in meters per second for a point on the tip of the propeller?

Find the angular velocity in radians per second for a particle that is moving along a circle with diameter 15 meters at a linear velocity of 20 meters per second.

What is the linear velocity in miles per hour of the tip of a 20-inch lawnmower blade that is rotating at 3000 rpms?

Find the angular velocity in radians per minute for a particle that is moving in a circular path at 95 mph on a circle with a radius of 8 inches.

**Velocity for Circular Motion:** For an object moving on a circular path of radius $r$, with constant angular velocity $\omega$, the linear velocity of the object is given by $v = r\omega$.

**Example:**
Any point on the surface of the earth (except at the poles) makes one revolution ($2\pi$ radians) about the axis of the earth in 24 hours. So the angular velocity of a point on the earth is $2\pi/24$ or $\pi/12$ radians per hour. The linear velocity of a point on the surface of the earth depends on its distance from the axis of the earth. What is the linear velocity in miles per hour of a point on the equator? (Use 3950 miles as the radius of the earth).